proprep™

# Workbook

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# Triple Integrals

## General Calculations with Triple Integrals

### Questions:

**1)** Compute the following integrals:

a. $\int_0^1\int_0^z\int_0^{x+z} 6xz\,dydxdz$

b. $\int_0^3\int_0^1\int_0^{\sqrt{1-z^2}} ze^y\,dxdzdy$

c. $\iiint_B xyz^2 dV$ , $B=\{(x,y,z)|0\le x\le 1,-1\le y\le 2,0\le z\le 3\}$

d. $\iiint_B 6xy dV$ , $B=\left\{(x,y,z)\middle|0\le x\le 1\ ,0\le y\le\sqrt{x}\ ,0\le z\le 1+x+y\ \right\}$

**2)** Compute the following integrals by changing the order of integration:

a. $\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,dxdydz$

b. $\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dydxdz$

c. $\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi e^{2x}\sin\pi y^2}{y^2}\,dxdydz$

d. $\int_0^2\int_0^{4-x^2}\int_0^x \frac{\sin 2z}{4-z}\,dydzdx$

**3)** Compute the volumes of the solids bounded by the given surfaces:

a. $y=0\ ,\ x=0\ ,\ x+y=1\ ,\ z=0,\ z=1+x+y$

b. $y=x^2, y=1,\ z=x^2+y^2\ ,\ z=0$

c. $y=\frac{2}{x}\ , y=2x\ , y=0.5x, z=x^2+y\ ,\ z=0\ \ (x\ge 0)$

d. $2y^2=x\ ,\ \frac{x}{4}+\frac{y}{2}+\frac{z}{4}=1\ ,\ z=0$

e. $x^2+\frac{y^2}{4}=1,\ z=y\ \ (z\ge 0)$

f. $x=0\ ,\ y=0\ , z=x+y,\ z=6$

**4)** Compute the mass and the center of mass of a cylinder whose height is $h$ and whose base has radius $r$. It is given that the density function $\delta$ at each point is proportional the distance of the point from the base, i.e. $\delta(x,y,z)=kz\ (k>0)$.

**5)** Compute the moment of inertia of the homogeneous* cube $V=\{(x,y,z)|0\le x\le a\ ,\ 0\le y\le b\ ,\ 0\le z\le c\}$ about the $z$-axis.
Express your answer in terms of the mass $M$ of the board.
* Homogeneous means having a constant density function.

## Answer Key:

**1)** a. 1 b. $\frac{1}{3}\left(e^3-1\right)$ c. $\frac{27}{4}$ d. $\frac{65}{28}$

**2)** a. $2\sin 4$ b. $3e-6$ c. $\frac{27}{4}$ d. $\frac{\sin^2 4}{2}$

**3)** a. $\frac{5}{6}$ b. $\frac{88}{100}$ c. $\frac{17}{6}$ d. $16\frac{1}{5}$
e. $\frac{8}{3}$ f. 36

**4)** $M=\frac{1}{2}kh^2\pi a^2$ , $(\bar{x},\bar{y},\bar{z})=\left(0,0,\frac{2h}{3}\right)$.

**5)** $\frac{2}{3}Ma^2$.

## Triple Integrals in Cylindrical and Spherical Coordinate Systems

### Questions:

**1)** Compute: $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{x^2+y^2} 21xy^2 dzdydx$.

**2)** Compute: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{1-x^2} \int_{\sqrt{x^2+y^2}}^{1} dzdydx$.

**3)** Switch to cylindrical coordinates (but don't compute): $\int_0^2 \int_0^{\sqrt{2x-2^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$.

**4)** A body is bounded by the cylinder $x^2+y^2=9$, the $xy$-plane below and the hemisphere $z=\sqrt{25-x^2-y^2}$ above.
Compute the volume of the body and its centroid.

**5)** Compute the volume and the centroid of the body bounded by the sphere $x^2+y^2+z^2=16$ above and by the cone $z=\sqrt{x^2+y^2}$.

**6)** Find the volume of the region above the $xy$ - plane bounded by the paraboloid $z=x^2+y^2$ and the cylinder $x^2+y^2=a^2$.

### Answer Key:

**1)** 4

**2)** $\frac{1}{3}\pi$

**3)** $\int\limits_{\theta=0}^{\pi/2} \int\limits_{r=0}^{2\cos\theta} \int\limits_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} rdzdrd\theta$

**4)** $\left(0,0,\frac{1107}{488}\right)$

**5)** $\left(0,0,\frac{3}{2\left(2-\sqrt{2}\right)}\right)$

**6)** $\frac{1}{2}a^4\pi$

# Triple Integrals and the Jacobian

## Questions:

**1)** Compute $\iiint\limits_{B} (z-y)^2 xydV$ when $B$ is body bounded by the surfaces $xy = 4$ , $xy = 2$ , $z = y+1$ , $z = y$ , $x = 3$ , $x = 1$ .

**2)** Compute the volume of the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ .

**3)** Compute $\iiint\limits_{E} x^2 dV$ when $E$ is the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ .

**4)** Compute the volume of the region bounded by the surfaces: $y = z^2$ , $y = 4z^2$ , $y = 4x$ , $y = 4x-12$ , $y = z$ , $y = 2z$

## Answer Key:

**1)** $2\ln 3$

**2)** $\frac{4}{3}\pi abc$

**3)** $\frac{4\pi}{15}a^2bc$

**4)** $\frac{105}{32}$