proprep™

Physics 1

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Variable Mass

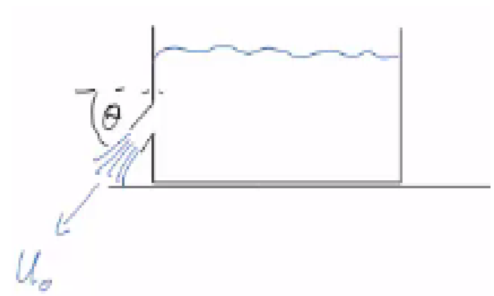
Using the Equation

Example:

1) Varying Mass and Friction.

A cart, with an initial mass m_0, is traveling on a surface with a coefficient of friction, μ_k.
At one end of the cart there is a hose which ejects water at a rate of α and at a velocity u_0.
The hose is at an S $\theta°$ to the x-axis.

a. Write an equation of motion.
b. Find the velocity as a function of time.



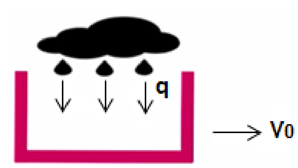
End of Chapter Questions

Questions:

2) Rain into a Cart.

A cart of mass M_0 is traveling at an initial velocity V_0.
At $t = 0$, rain falling perpendicularly to the ground starts filling the cart at a rate q.

a. What is the velocity of the cart as a function of time?
b. When the cart reaches a mass of M_f the rain stops.
   What is the velocity of the cart after the rain stops?



3) Balloon.

A balloon of mass M is filled with gas. $\frac{3}{4}$ of the balloon's mass is the mass of the gas.
The balloon is released from rest and the gas exits the balloon at a velocity of u relative to the balloon.
The balloon accelerates upwards along a straight line at an acceleration of $0.5g$.

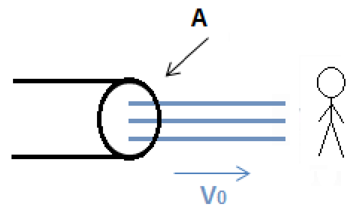
a. At what rate is the gas emitted?
b. What is the maximum height the balloon will reach?

4) Hose Spraying on Person.

A hose sprays water on a person. The cross-sectional area of the hose is A and the density of the water is ρ.

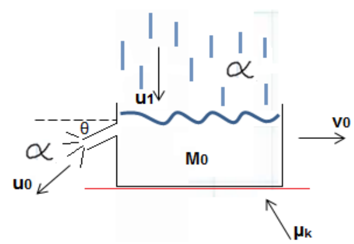
The velocity with which the water exits the hose is v_0.

a. Find the force acting on the stationary person being sprayed, given that no water is sprayed back.
b. Find the force acting on the person who is running away at a velocity $v < v_0$.



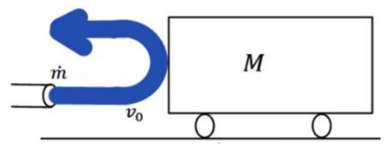
5) A Cart with Accretion Ejection and Friction.

a. Find the cart's equation of motion.
b. What is vf of the cart?
c. What is the velocity of the cars as a function of time?



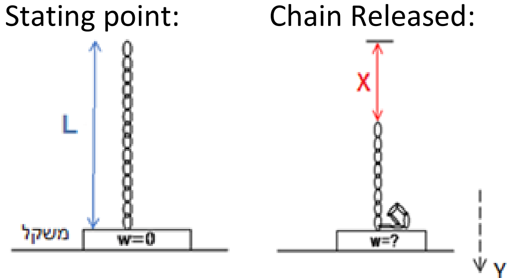
6) Elastic Collision - Water Sprayed on a Cart.

A cart of mass M is on a flat, frictionless plane.
A hose sprays water on the cart, driving the cart forwards. The water exits the hose at a rate of $\frac{dm}{dt}$, with a velocity of v_0. The collision between the water and the cart is elastic.
What is the velocity of the cart, as a function of time?



7) Chain Falling on Scale

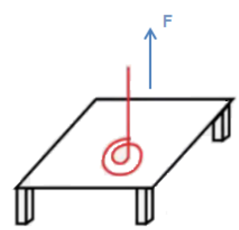
A chain of length L and mass M is held vertically above a scale, such that the bottom end is just touching the scale. The chain is then released from rest.
Find the weight shown on the scale as a function for x (the distance which the top end of the chain fell).



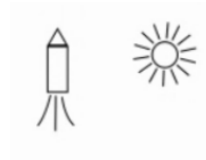
8) Lifting a String off the Table.

A string of mass m and length l is resting on a table.
A force, F, begins lifting the string, at a velocity of V_0.

a. What is the force as a function of time?
b. How much energy is wasted as a function of the distance travelled?



9) Rocket Orbiting a Star.

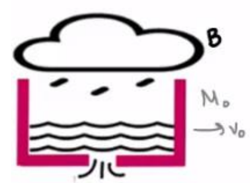


A rocket of mass m_0 orbits a star of mass M, a distance of r_0 away.
The rocket is moving at a velocity of v_0.
At time $t=0$, the rocket emits gas at a relative velocity of u and at a rate of α.
Find the equation of motion of the rocket in the radial direction.

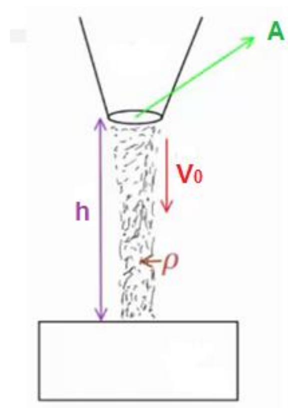
10) Rain at an Angle.

Cart A has an initial mass, M_0, and an initial velocity of v_0 in the right direction.
Rain falls in the cart at $S\,\alpha°$ and velocity u such that the cart fills up at a rate of q.
Cart B has the same starting mass as A and the same initial velocity as A.
Cart B is already filled with rain water.
There is a hole in the cart and water is let out at the same rate as water enters.
Write an equation for the velocity of each cart as a function of time.

$$\sum Fext = M\frac{dv}{dt} + \left|\frac{dm}{dt}\right| u(-1)$$

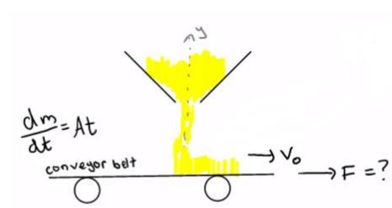


11) Funnel and Scales



a. How much sand exits through the funnel per second?
b. What is the velocity of the sand when it hits the scales?
c. Whilst the sand is falling, when the scales show W, what is the ratio between the real weight of the sand on the scale. Compared to the indicated mass on the scale?
d. When a weight of W is shown on the scale, the funnels opening is shut.
After some moments, what weight will the scale measure?
e. The scale is now accelerated upwards at $a = 5\frac{\text{m}}{\text{s}^2}$.
What weight will the scale show after some time?

12) Sand on a Conveyor Belt



a. What force, F, would we need, in order for the conveyor belt to travel at a constant velocity?
b. How much energy is invested per second?

Answer Key:

1) a. $\mu_k\left(\left(\mu_0-\alpha t\right)g-u_0\sin\theta\alpha\right)=\left(\mu_0-\alpha t\right)\dfrac{dv_x}{dt}-\mu_0\cos\theta\alpha$ b. $v\left(t\right)=-\mu_k gt-\dfrac{c}{\alpha}\ln\left(\dfrac{M_0-\alpha t}{M_0}\right)$

2) a. $v\left(t\right)=\dfrac{M_0V_0}{M_0+qt}$ b. $v_f=\dfrac{M_0V_0}{M_f}$

3) a. $\dfrac{dm}{dt}=\dfrac{-3g}{2u_0}Me^{\frac{-3g}{2u_0}t}$ b. $\Delta y_2=\dfrac{1}{2g}\left(\dfrac{u_0}{3}\ln 4\right)^2+\dfrac{g}{4}\left(\dfrac{2u_0}{3g}\ln 4\right)^2$

4) a. $\sum F_{person}=\rho A v_0^2$ b. $\sum F_{person}=\rho A\left(v_0-v\right)^2$

5) a. $-\mu_k N=M_0\dfrac{dv}{dt}+\alpha v\left(t\right)-u_0\alpha\cos\theta$ b. $v_f=\left(u_0\alpha\cos\theta-\mu_k N\right)\dfrac{1}{\alpha}$

c. $v\left(t\right)=\dfrac{-1}{\alpha}e^{-\frac{\alpha}{M_0}t}\left[\left(c-v_0\right)-c\right]$

6) $v\left(t\right)=v_0\left(1-\dfrac{M}{2\dot{m}t+M}\right)$

7) $N\left(x\right)=\dfrac{3M}{L}xg$

8) a. $F=v_0^2\dfrac{m}{L}v_0t\dfrac{m}{L}g$ b. $\Delta E_y=\dfrac{1}{2}v_0^2\dfrac{m}{L}y$

9) $\dfrac{-mM}{r^2}m\left(\ddot{r}-\dfrac{\left(v_0-u\ln\dfrac{\left(m_0-\alpha t\right)}{m_0}\right)^2}{r^2}\right)$

10) $A:\ v\left(t\right)=\dfrac{M_0\left(u\cos\alpha+v_0\right)}{M_0+qt}-u\cos\alpha,\quad B:\ v\left(t\right)=\left(u\cos\alpha+v_0\right)e^{-\frac{qt}{M_0}}-u\cos\alpha$