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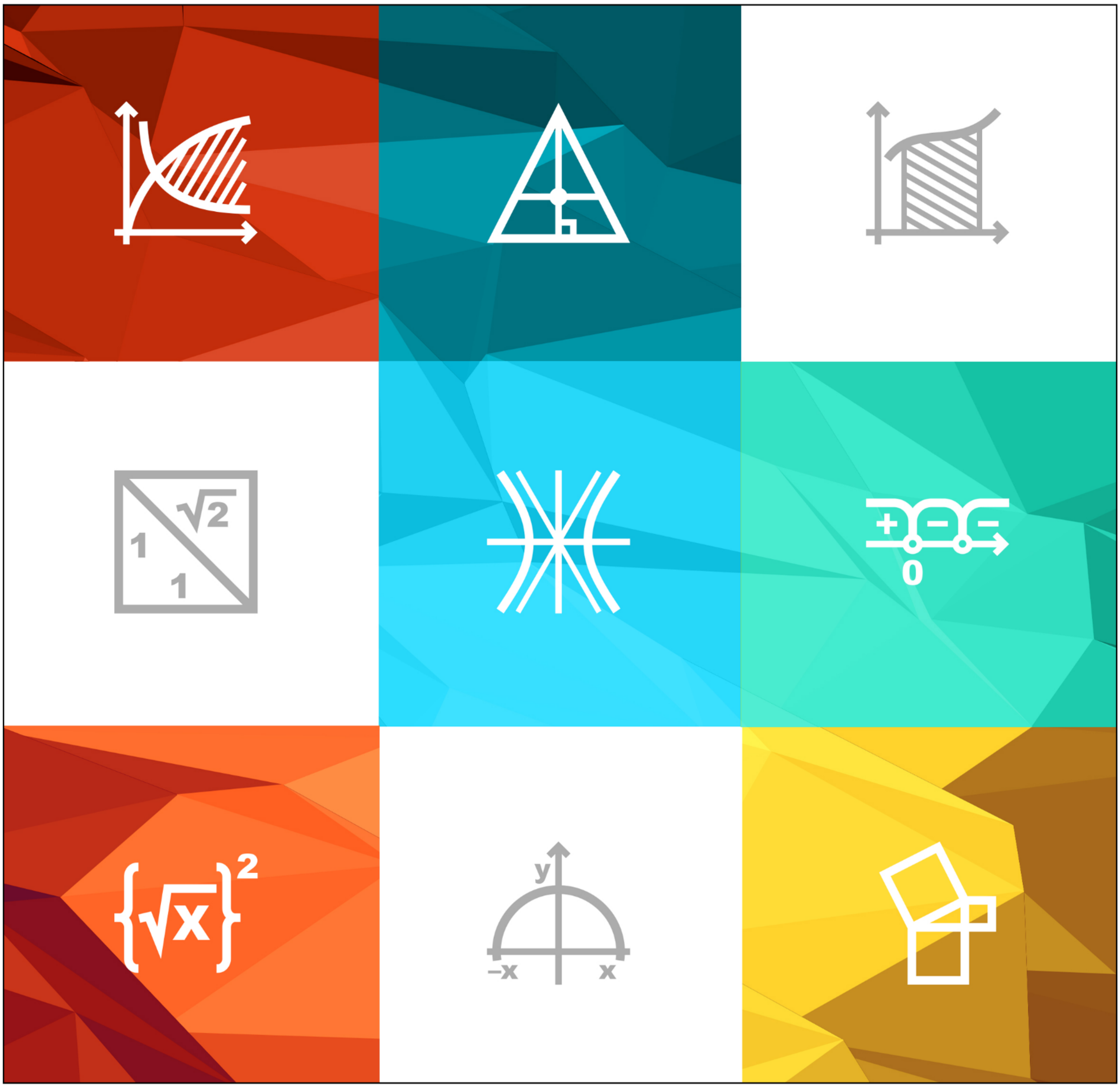
Workbook



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Vector Spaces

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Questions:

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2) Check if W is a subspace of $\mathbb{R}^3$, where $W = \{\langle a,b,c\rangle \mid a=c\}$.

3) Check if W is a subspace of $\mathbb{R}^3$, where $W = \{\langle a,b,c\rangle \mid a=3b\}$.

4) Check if W is a subspace of $\mathbb{R}^3$, where $W = \{\langle a,b,c\rangle \mid a<b<c\}$.

5) Check if W is a subspace of $\mathbb{R}^3$, where $W = \{\langle a,b,c\rangle \mid a=c^2\}$.

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Answer Key:

1) W is a subspace of $\mathbb{R}^3$.

2) W is a subspace of $\mathbb{R}^3$.

3) W is a subspace of $\mathbb{R}^3$.

4) W is not a subspace of $\mathbb{R}^3$.

5) W is not a subspace of $\mathbb{R}^3$.

6) W is a subspace of $\mathbb{R}^3$.

7) W is not a subspace of $\mathbb{R}^3$.

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Liner Combinations and Span in R^n

Questions:

1) We are given the following vectors in $\mathbb{R}^4$: $u_1 = \langle 4,1,1,5 \rangle$, $u_2 = \langle 0,11,-5,3 \rangle$, $u_3 = \langle 2,-5,3,1 \rangle$, $u_4 = \langle 1,3,-1,2 \rangle$.

a. Is u_1 a linear combination of u_4?

b. Does u_1 belong to $Sp\{u_4\}$?

c. Is the set $\{u_1, u_4\}$ linearly dependent?

2) We are given the following vectors in $\mathbb{R}^4$: $u_1 = \langle 4,1,1,5 \rangle$, $u_2 = \langle 0,11,-5,3 \rangle$, $u_3 = \langle 2,-5,3,1 \rangle$, $u_4 = \langle 1,3,-1,2 \rangle$.

a. Is u_3 a linear combination of u_1 and u_2?

b. Does u_3 belong to $Sp\{u_1, u_2\}$?

c. Is the set $\{u_1, u_2, u_3\}$ linearly dependent?
If so, try to write each vector in the set as a linear combination of the others.

3) We are given the following vectors in $\mathbb{R}^4$: $u_1 = \langle 4,1,1,5 \rangle$, $u_2 = \langle 0,11,-5,3 \rangle$, $u_3 = \langle 2,-5,3,1 \rangle$, $u_4 = \langle 1,3,-1,2 \rangle$.

a. Is u_4 a linear combination of u_1 and u_2?

b. Does u_4 belong to $Sp\{u_1, u_2\}$?

c. Is the set $\{u_1, u_2, u_4\}$ linearly dependent?
If so, try to write each vector in the set as a linear combination of the others.

4) We are given the following vectors in $\mathbb{R}^4$: $u_1 = \langle 4,1,1,5 \rangle$, $u_2 = \langle 0,11,-5,3 \rangle$, $u_3 = \langle 2,-5,3,1 \rangle$, $u_4 = \langle 1,3,-1,2 \rangle$,

and $v = \langle 4,12,k,-2k \rangle$.

a. What should the value of k be, in order for the vector v to be a linear combination of u_1 and u_2?

b. What should the value of k be, in order for the vector v to belong to $Sp\{u_1, u_2\}$?

c. What should the value of k be in order for the set $\{u_1, u_2, v\}$ to be linearly dependent?

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5) We are given the following vectors in $\mathbb{R}^4$: $u_1 = \langle 4,1,1,5 \rangle$, $u_2 = \langle 0,11,-5,3 \rangle$, $u_3 = \langle 2,-5,3,1 \rangle$, $u_4 = \langle 1,3,-1,2 \rangle$,

and $v = \langle a,b,c,d \rangle$.

a. What are the conditions on a,b,c,d , in order for v to be a linear combination of u_1 and u_2 ?

b. What are the conditions on a,b,c,d , in order for v to belong to $Sp\{u_1,u_2\}$?

c. What are the conditions on a,b,c,d , in order for the set $\{u_1,u_2,v\}$ to be linearly dependent?

Answer Key:

1) a. No b. No c. No

2) a. Yes. $u_3 = \frac{1}{2}(u_1 - u_2)$ b. Yes. Follows from a.

c. Yes $u_1 = 2u_3 + u_2$, $u_2 = u_1 - 2u_3$

3) a. Yes. $u_4 = \frac{1}{4}(u_1 + u_2)$ b. Yes. Follows from a.

c. Yes. $u_1 = 4u_4 - u_2$, $u_2 = 4u_4 - u_1$

4) a-c. $k = -4$

5) a+b. Refer to the video.

c. $\begin{cases} a = 5t + 3s \\ b = 4t - 13s \\ c = 7s \\ d = 7t \end{cases}$

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Basis for R^n

Questions:

1) Check if each of the following sets is a basis of $\mathbb{R}^3$:

a. $\{\langle 1,0,1\rangle,\langle 0,0,1\rangle\}$

b. $\{\langle 1,1,2\rangle,\langle 1,2,3\rangle,\langle 3,3,4\rangle,\langle 2,2,1\rangle\}$

c. $\{\langle 1,2,3\rangle,\langle 4,5,6\rangle,\langle 7,8,9\rangle\}$

2) Check if each of the following sets is a basis of $M_{2\times 2}[\mathbb{R}]$ (AKA $M_2[\mathbb{R}]$):

a. $\left\{\begin{bmatrix}1&2\\3&4\end{bmatrix},\begin{bmatrix}5&6\\7&8\end{bmatrix},\begin{bmatrix}9&1\\2&3\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}1&2\\3&4\end{bmatrix},\begin{bmatrix}5&6\\7&8\end{bmatrix},\begin{bmatrix}9&1\\2&3\end{bmatrix},\begin{bmatrix}5&6\\7&2\end{bmatrix},\begin{bmatrix}5&16\\7&8\end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix}1&1\\1&1\end{bmatrix},\begin{bmatrix}0&1\\1&1\end{bmatrix},\begin{bmatrix}0&0\\1&1\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$

3) Check if each of the following sets is a basis of $P_2[\mathbb{R}]$ (deg ≤ 2 poly):

a. $\{1+x,\ x^2+2x+3\}$

b. $\{1+x,\ x^2+2x+3, 2x+4x^2, x-x^2\}$

c. $\{1+2x+3x^2,\ 4+5x+6x^2, 7+8x+10x^2\}$

4) Consider the following set of vectors in $\mathbb{R}^3$: $T=\{\langle 1,2,3\rangle,\ \langle 4,5,6\rangle,\ \langle 7,8,9\rangle,\langle 2,3,4\rangle\}$.

a. Is T a basis of $\mathbb{R}^3$?

b. Find a maximal linearly independent subset T' of T.

c. Extend T to a basis of $\mathbb{R}^3$.

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Answer Key:

1) a. The two vectors can't form a basis for $\mathbb{R}^3$.

b. The four vectors can't form a basis for $\mathbb{R}^3$.

c. The vectors are linearly dependent and so are not a basis for $\mathbb{R}^3$.

2) a. The three vectors can't form a basis for $M_{2\times2}[\mathbb{R}]$.

b. The five vectors can't form a basis for $M_{2\times2}[\mathbb{R}]$.

c. The four vectors do form a basis for $M_{2\times2}[\mathbb{R}]$.

3) a. The two vectors can't form a basis for $P_2[\mathbb{R}]$.

b. The four vectors can't form a basis for $P_2[\mathbb{R}]$.

c. The three vectors do form a basis for $P_2[\mathbb{R}]$.

4) a. No, since it contains more than three vectors.

b. $T' = \{\langle 1,2,3\rangle, \langle 4,5,6\rangle\}$

c. $T^* = \{\langle 1,2,3\rangle, \langle 4,5,6\rangle, \langle 0,0,1\rangle\}$

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Basis for a Solution Space, Homogeneous SLE A in R^n

Questions:

1) Here are 3 systems of linear equations (SLEs):

1. $\begin{cases} x+y-z+2w=0 \\ 3x-y+7z+4w=0 \\ -5x+3y-15z-6w=0 \end{cases}$ 2. $\begin{cases} x-y+z+w=0 \\ x+2z-w=0 \\ x+y+3z-3w=0 \end{cases}$ 3. $\begin{cases} x-y+z+w=0 \\ 2x-2y+2z+2w=0 \end{cases}$

Let's denote by W, U and V the subspaces spanned by 1., 2. and 3., respectively.

a. Find a basis for each of W, U and V as well as their dimension.

b. i. Find a basis for $U+V$ and its dimension.

ii. Find the dimension of $U \cap V$.

c. Find a basis for $U \cap V$.

2) Let $U=\{\langle a,b,c,d\rangle \in \mathbb{R}^4 \mid a=c, b=d\}$. Find a basis and the dimension of U.

3) Let $U=\{\langle a,b,c,d\rangle \in \mathbb{R}^4 \mid c=a+b, d=b+c\}$. Find a basis and the dimension of U.

4) Let $U=\{v \in \mathbb{R}^4 \mid v\cdot\langle 1,-1,1,-1\rangle = 0\}$. Find a basis and the dimension of U.

Answer Key:

1) a. $B_W=\{\langle -1.5,2.5,1,0\rangle, \langle -1.5,-0.5,0,1\rangle\}$ $\dim W=2$,

$B_U=\{\langle -2,-1,1,0\rangle, \langle 1,2,0,1\rangle\}$ $\dim U=2$,

$B_V=\{\langle -1,0,0,1\rangle, \langle -1,0,1,0\rangle, \langle 1,1,0,0\rangle\}$ $\dim V=3$

b. i. $B_{U+V}=\{\langle 0,0,-1,1\rangle, \langle 0,1,1,0\rangle, \langle 1,1,0,0\rangle\}$ $\dim U+V=3$

ii. $2=\dim(U\cap V)$

c. $B_{U\cap V}=\{\langle -2,-1,1,0\rangle, \langle 1,2,0,1\rangle\}$

2) $B_U=\{\langle 0,1,0,1\rangle,\langle 1,0,1,0\rangle\}$, $\dim U=2$

3) $B_U=\{\langle -1,1,0,1\rangle,\langle 2,-1,1,0\rangle\}$, $\dim U=2$

4) $B_U=\{\langle 1,0,0,1\rangle,\langle -1,0,1,0\rangle,\langle 1,1,0,0\rangle\}$ $\dim U=3$

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Basis of a Subspace of R^n

Questions:

1) Consider the subspace of $\mathbb{R}^4$ defined as follows:
$U = span\{\langle 1,1,-1,2\rangle, \langle 3,-1,7,4\rangle, \langle -5,3,-15,-6\rangle\}$.

a. Find a basis for U and its dimension.
b. Find a homogeneous SLE whose solution space is U.

2) Consider the subspace of $\mathbb{R}^4$ defined as follows:
$V = span\{\langle 1,1,-1,1\rangle, \langle 1,0,2,-1\rangle, \langle 1,1,3,-3\rangle, \langle 5,1,5,8\rangle\}$.
Find, for V: a basis, the dimension, a homogeneous SLE.

3) Consider the two subspaces of $\mathbb{R}^4$ defined as follows:
$U = span\{\langle 1,1,-1,2\rangle, \langle 3,-1,7,4\rangle, \langle -5,3,-15,-6\rangle\}$
$V = span\{\langle 1,-1,1,1\rangle, \langle 1,0,2,-1\rangle, \langle 1,1,3,-3\rangle, \langle 5,1,5,8\rangle\}$.
Find a basis and the dimension of $U+V$.

4) Consider the two subspaces of $\mathbb{R}^4$ defined as follows:
$U = span\{\langle 1,1,-1,2\rangle, \langle 3,-1,7,4\rangle, \langle -5,3,-15,-6\rangle\}$
$V = span\{\langle 1,-1,1,1\rangle, \langle 1,0,2,-1\rangle, \langle 1,1,3,-3\rangle, \langle 5,1,5,8\rangle\}$.
Find a basis and the dimension of $U \cap V$.

Answer Key:

1) a. $B_U = \{\langle 1,1,-1,2\rangle, \langle 0,-4,10,-2\rangle\}$, $\dim U = 2$ b. $\begin{cases} -3x+5y+2z=0 \\ -3x-y+2t=0 \end{cases}$

2) $-8x-y+5z+2t=0$, $B_V = \{\langle 1,-1,1,1\rangle, \langle 0,1,1,-2\rangle, \langle 0,0,-2,5\rangle\}$, $\dim V = 3$

3) $B_{U+V} = \{\langle 1,1,-1,2\rangle, \langle 0,-4,10,-2\rangle, \langle 0,0,1,0\rangle, \langle 0,0,0,1\rangle\}$, $\dim U+V = 4$

4) $\dim U \cap V = 1$, $B_{U\cap V} = \{\langle 5,1,5,8\rangle\}$

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Basis for Row and Column Spaces

Questions:

1) Find a basis and the dimension of the row space and the column space of the following matrix. What is its rank?

$$\begin{bmatrix} 4 & 1 & 1 & 5 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 1 & 3 & -1 & 2 \end{bmatrix}$$

2) Find a basis and the dimension of the row space and the column space of the following matrix. What is its rank?

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 5 \\ 2 & 5 & 3 & 1 & 6 \\ 1 & -1 & -2 & 2 & 1 \\ -2 & 3 & 5 & -4 & -1 \end{bmatrix}$$

Answer Key:

1) $B_{row} = \{\langle 4,1,1,5\rangle, \langle 0,11,-5,3\rangle\}$ $\dim(row) = 2$; $B_{col} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -2 \\ 1 \end{bmatrix} \right\}$, $\dim(col) = 2$;

rank=2.

2) $B_{row} = \{\langle 1,2,1,3,5\rangle, \langle 0,1,1,-5,-4\rangle, \langle 0,0,0,1,1\rangle\}$; $\dim(row) = 3$.

$B_{col} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -16 \\ 37 \end{bmatrix} \right\}$; $\dim(col) = 3$; Rank=3.

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Vector spaces

Questions:

Check if W is a subspace of $M_n[\mathbb{R}]$, where:

1) $W=\{A|\ A=A^T\}$.

2) W is the set of matrices which *commute* with a given matrix B. That is, $W=\{A|\ AB=BA\}$.

3) W is the set of matrices whose determinant is 0. That is, $W=\{A|\ \det A=0\}$.

4) W is the set of matrices which are equal to their own square. That is, $W=\{A|\ A^2=A\}$.

5) W is the set of upper-triangular matrices.

6) W is the set of matrices whose product with a given matrix B is 0. That is, $W=\{A|\ AB=\mathbf{0}\}$.

7) W is the set of matrices whose trace is 0. That is, $W=\{A|\ tr(A)=0\}$.

8) W is the set of matrices such that the sum of each row is 0.

Check if W is a subspace of $P_n[\mathbb{R}]$, where:

9) W consists of the polynomials having 4 as a root. I.e., $W=\{p(x)|\ p(4)=0\}$.

10) W consists of the polynomials with degree ≤ 4. I.e., $W=\{p(x)|\ \deg(p)\le 4\}$.

11) W consists of the polynomials with integer coefficients.

12) W consists of the polynomials with only even powers of x in its terms.

13) W consists of the polynomials having degree n where $4\le n\le 7$.

14) $W=\{p(x)|p(0)=1\}$.

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Check if W is a subspace of $F[\mathbb{R}]$, where:

15) W consists of all even functions. I.e., $W = \{ f(x) \mid f(x) = f(-x) \text{ for all } x \in \mathbb{R} \}$.

16) W consists of all bounded functions. I.e.,
$W = \{ f(x) \mid |f(x)| \le M \text{ for all } x \in \mathbb{R}, \text{ for some } M > 0 \}$.

17) W consists of all continuous functions.

18) W consists of all differentiable functions.

19) W consists of all constant functions.

20) $W = \left\{ f(x) \mid \int_0^1 f(x)\,dx = 4 \text{ (assume } f \text{ is integrable)} \right\}$.

21) $W = \{ f(x) \mid f'(x) = 0 \text{ (assume } f \text{ is differentiable)} \}$.

22) $W = \{ f(x) \mid f'(x) = 1 \text{ (assume } f \text{ is differentiable)} \}$.

23) $W = \{ f \mid f(x+1) = f(x) \text{ for all } x \in \mathbb{R} \}$.

Check if W is a subspace of $\mathbb{C}^3[\mathbb{R}]$:

24) Check if W is a subspace of $\mathbb{C}^3[\mathbb{R}]$, where $W = \{ \langle z_1, z_2, z_3 \rangle \mid z_2 = \overline{z}_1, z_3 = z_1 + \overline{z}_1 \}$.

25) Check if $W = \{ \langle z_1, z_2, z_3 \rangle \mid z_2 = \overline{z}_1, z_3 = z_1 + \overline{z}_1 \}$
is a subspace of $\mathbb{C}^3$ (over the complex field $\mathbb{C}$).

Answer Key:

1)-2) Is a subspace

3)-4) Not a subspace

5)-10) Not a subspace

11) Not a subspace

12) Is a subspace

13)-14) Not a subspace

15)-19) Is a subspace

20) Not a subspace

21) Is a subspace

22) Not a subspace

23)-24) Is a subspace

25) Not a subspace

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Linear Dependence

Questions:

1) We are given the following matrices in $M_2[\mathbb{R}]$:

$A=\begin{bmatrix}4 & 1\\ 1 & 5\end{bmatrix}$, $B=\begin{bmatrix}0 & 11\\ -5 & 3\end{bmatrix}$, $C=\begin{bmatrix}2 & -5\\ 3 & 1\end{bmatrix}$, $D=\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}$

a. Are these matrices linearly dependent?
b. If so, try to write each of them as a linear combination of the rest.
c. Does A belong to $Sp\{B,C\}$?

2) We are given the following polynomials in $P_3[\mathbb{R}]$:

$p_1(x)=4+x+x^2+5x^3$, $p_2(x)=11x-5x^2+3x^3$,
$p_3(x)=2-5x+3x^2+x^3$, $p_4(x)=1+3x-x^2+2x^3$

a. Are these polynomials linearly dependent?
b. If so, try to write each of them as a linear combination of the rest.
c. Does p_2 belong to $Sp\{p_1,p_3\}$?

3) We are given the following set of vectors in $\mathbb{R}^3$: $S=\{\langle c,2,4\rangle,\langle 2.4,a,2\rangle,\langle c,b,6\rangle,\langle b,2,a\rangle\}$
For which values of a,b,c is S linearly dependent?

4) We are given that the set $\{u,v,w\}$ of vectors is linearly independent in $V[F]$.

a. Is the set $\{u-v,u-w,u+v-2w\}$ linearly dependent?
b. If so, try to write each vector in the set as a linear combination of the others.

5) We are given that the set $\{u,v,w\}$ of vectors is linearly independent in $V[F]$.

a. Is the set $\{u+v,v+w,w\}$ linearly dependent?
b. If so, try to write vector in the set as a linear combination of the others.

6) We are given that the set $\{u,v,w\}$ of vectors is linearly independent in $V[F]$.

a. Is the set $\{u+2v+3w,4u+5v+6w,7u+8v+9w\}$ linearly dependent?
b. If so, try to write each vector in the set as a linear combination of the others.

7) Is the set of vectors $\{\langle 1,i,i-1\rangle,\langle i+1,i-1,-2\rangle\}$ linearly independent in $\mathbb{C}^3[\mathbb{C}]$?

8) Is the set of vectors $\{\langle 1,i,i-1\rangle,\langle i+1,i-1,-2\rangle\}$ linearly independent in $\mathbb{C}^3[\mathbb{R}]$?

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Answer Key:

1) a. Yes, they're linearly dependent.

b. $A = B + 2C$, $B = A - 2C$, $C = \frac{1}{2}A - \frac{1}{2}B$, $D = \frac{1}{4}A + \frac{1}{4}B$

c. Yes, follows from $A = B + 2C$.

2) a. Yes, they are linearly dependent.

b. $p_1 = p_2 + 2p_3$, $p_2 = p_1 - 2p_3$, $p_3 = \frac{1}{2}p_1 - \frac{1}{2}p_2$, $p_4 = \frac{1}{4}p_1 + \frac{1}{4}p_2$

c. Yes, follows from $p_2 = p_1 - 2p_3$.

3) For all values a, b, c S linearly is dependent.

4) a. Yes, they are linearly dependent.

b. $x = 2y - z$
$y = 0.5x + 0.5z$
$z = 2y - x$

5) a. No

b. N/A

6) a. Yes, they are linearly dependent.

b. $x = 2y - z$
$y = 0.5x + 0.5z$
$z = 2y - x$

7) No, the vectors are linearly dependent.

8) The vectors are linearly independent.

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Basis for Known Vector Spaces

Questions:

1) Check if each of the following sets is a basis of $M_{2\times2}[\mathbb{R}]$ (A.K.A $M_2[\mathbb{R}]$):

a. $\left\{\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix},\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix},\begin{bmatrix}9 & 1\\2 & 3\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix},\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix},\begin{bmatrix}9 & 1\\2 & 3\end{bmatrix},\begin{bmatrix}5 & 6\\7 & 2\end{bmatrix},\begin{bmatrix}5 & 16\\7 & 8\end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix},\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right\}$

2) Check if each of the following sets is a basis of $P_2[\mathbb{R}]$ (deg ≤ 2 poly):

a. $\{1+x,\ x^2+2x+3\}$

b. $\{1+x,\ x^2+2x+3, 2x+4x^2, x-x^2\}$

c. $\{1+2x+3x^2,\ 4+5x+6x^2, 7+8x+10x^2\}$

Answer Key:

1) a. No, the three vectors can't form a basis.
b. No, the five vectors can't form a basis.
c. Yes, the four vectors do form a basis.

2) a. No, the two vectors can't form a basis.
b. No, the four vectors can't form a basis.
c. Yes, the three vectors do form a basis.

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Basis for a Solution Space, Homogeneous SLE

Questions:

1) Let $U = \{A \in M_2[\mathbb{R}] \mid A = A^T\}$. Symmetric 2x2 matrices.
Find a basis and the dimension of U.

2) Let $U = \left\{A \in M_2[R] \mid A \cdot \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\}$.
Find a basis and the dimension of U.

3) Let $U = \{p(x) \in P_3[\mathbb{R}] \mid p(1) = 0\}$
Find a basis and the dimension of U.

Answer Key:

1) $B_U = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\}$, $\dim U = 3$.

2) $U = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\}$, $\dim U = 0$, $B_U = \varnothing$ [empty set].

3) $B_U = \{p_1(x) = -1 + x^3 \;\; p_2(x) = -1 + x^2, p_3(x) = -1 + x\}$, $\dim U = 3$

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Basis of a Subspace of a Known Vector Space

Questions:

1) Consider the subspace of $M_2[\mathbb{R}]$ defined as follows:

$$U = span\left\{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}\right\}.$$

Find a basis and the dimension of U .

2) Consider the subspace of $P_3[\mathbb{R}]$ defined as follows:

$$U = span\{1+x-x^2+2x^3, 4+x-x^2+x^3, 2-x+x^2-3x^3\}$$

Find a basis and the dimension of U .

Answer Key:

1) $B_U = \left\{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -3 \\ 3 & -7 \end{bmatrix}\right\}, \ \dim U = 2$

2) $B_U = \{1+x-x^2+2x^3, -3x+3x^2-7x^3\}, \ \dim U = 2$

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Coordinate Vectors and Change of Basis

Questions:

1) Given the following two bases of $P_2[\mathbb{R}]$:

$B_1 = \{1+x, x, x+x^2\}$; and $B_2 = \{1+x^2, x+x^2, x^2\}$,

and let $p(x) = a+bx+cx^2$, be a general polynomial in $P_2[\mathbb{R}]$.

Compute $[p(x)]_{B_1}$, the coordinate vector of $p(x)$ relative to B_1 and B_2.

Find the change-of-basis matrix from B_1 to B_2.

2) Given the following two bases of $M_2[\mathbb{R}]$:

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

Compute the coordinate vector of $X = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$ relative to B and E.

Find the change-of-basis matrix from B to E.

Answer Key:

1) $[v]_{B_1} = \langle a, b-a-c, c \rangle$; $\quad [v]_{B_2} = \langle a, b, c-a-b \rangle$

$$[M]_{B_1}^{B_2} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

2) $[X]_B = \langle x, \mathrm{y-x}, z-y+x, t-z+y-x \rangle$

E is the elementary or standard basis of $M_2[\mathbb{R}]$:

$[X]_E = \langle x, \mathrm{y}, z, t \rangle$

The change-of-basis matrix from B to E: $[M]_B^E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}$

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