

# Workbook



Table of Content:

Graphical and Numerical Methods .....2  
Graphical and Numerical Methods: .....2

# Graphical and Numerical Methods

## Graphical and Numerical Methods:

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### Questions:

- 1) Given the differential equation  $y' = x + y$ 
  - a. Sketch line elements for the points  $(x, y)$  where  $x, y \in \{-2, -1, 0, 1, 2\}$
  - b. Compare with a computer-aided sketch of the direction field.
  
- 2) Given the differential equation  $y' = x + y$ 
  - a. Describe the isoclines associated with this DE and sketch a few (say, 3).
  - b. Draw a few line elements on each of these.
  - c. Compare with a computer-aided sketch of the direction field.
  
- 3) Given the differential equation  $y' = x + y$ 
  - a. Show a computer-aided sketch of the associated direction field.
  - b. Use this to manually sketch the solutions to the DE passing through the points  $(0, 0)$ ,  $(0, 1)$  and  $(0, -1)$ .
  
- 4) Given the differential equation  $y' = x^2 + y^2 - 2$ 
  - a. Describe the isoclines associated with this DE and sketch those for slopes  $c = -1, 0, 2$ .
  - b. Draw a few line elements on each of these.
  - c. Compare with a computer-aided sketch of the direction field.
  - d. Use the DF to manually sketch the solutions to the DE passing through the points  $(0, 0)$ ,  $(0, 2)$  and  $(0, -2)$ .
  
- 5) Given the IVP  $y' = x + y$ ;  $y(0) = 0$ 
  - a. Estimate  $y(1)$  using Euler's method with step-size  $h = 0.25 = \frac{1}{4}$ .  
[Manual computation].
  - b. Estimate  $y(1)$  using Euler's method with step-size  $h = 0.05 = \frac{1}{20}$ .  
[Computer-aided].
  - c. As above but with  $h = 0.01 = \frac{1}{100}$ .
  - d. Verify that  $y = e^x - x - 1$  is the solution to the IVP and compute  $y(1)$ .
  - e. Round the results to 3 decimal places.  
What can we say here about step-size and accuracy?

### Answer Key:

For complete solutions please refer to the videos.