proprep™

Physics 1

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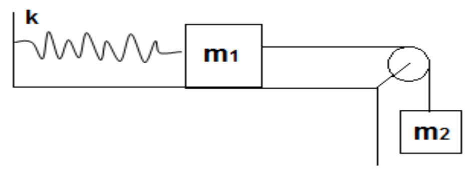
Harmonic Motion-Oscillations

Simple Harmonic Motion

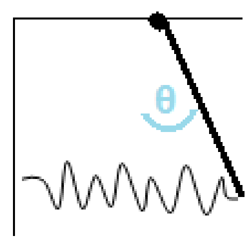
Questions:

1) **Mass on a Table Attached to Hanging Mass.**
Mass m_1 is placed on a frictionless table and is attached to a spring of constant k.
A string, which is attached to the mass, is passed along an ideal pulley and attached to a further mass, m_2, which hangs in the air.



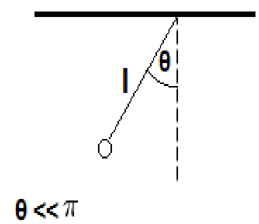
   a. What is the system's point of equilibrium? (Define the origin as the point where the spring is loose)
   b. Find the system's frequency of oscillation.
   c. What is the maximal amplitude possible for the motion such that the tension in the string will not reset during the course of the motion.

2) **Hanging Rod attached to a Spring.**
A rod, of length L and mass M, hangs from the ceiling and is free to rotate about its point of attachment.
The other end of the rod is attached via a spring, of constant K, to the wall.

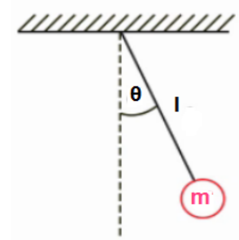


   a. Show that the motion of the rod, at small angles, is harmonic, and find ω.
   b. Find the angle of the rod as a function of time, if the rod is released from rest at an angle of θ_0°.

3) **Mathematical Pendulum (torque).**
A pendulum is attached to the ceiling. The length of the rope is l.
Find the frequency of small-angle oscillations, as well as the angle as a function of time.
The pendulum begins its movement from rest at an initial angle of θ°.

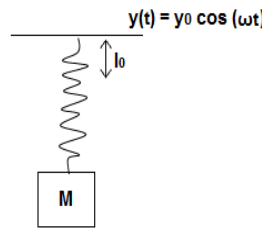


4) Physical Pendulum.
Find the frequency of a disk of mass m and radius R, which is attached to a ceiling by a string of mass M and length l.



Driven Harmonic Motion

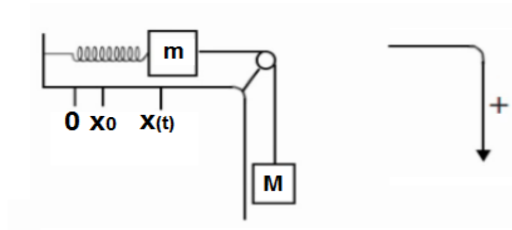
5) Mass attached To Moving Plank.
Mas M is attached via a spring to a horizontal board which moves in the y direction, according to $y(t) = y_0 \cos(\omega t)$.
The spring constant is k and its length is l_0.
Find the position of the mass as a function of time.



End of Chapter Questions

6) Two Masses and a Spring.

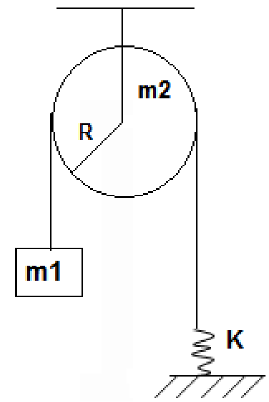
We are given the system in the following diagram.



a. What is the frequency of the system?

b. At $t = 0$ the mass was released from rest after being pulled a distance D.
Find the solution to the equation of motion.

7) Pulley Mass and a Spring.



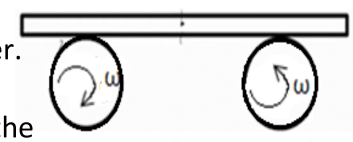
Mass m_1 is tied via a string, over an unideal pulley system, to a spring (of constant k) which is attached to the ground.
The radius and mass of the pulley is R and m_2.
The string does not slip on the pulley.

a. Where is the point of equilibrium?

b. What is the frequency of oscillations?

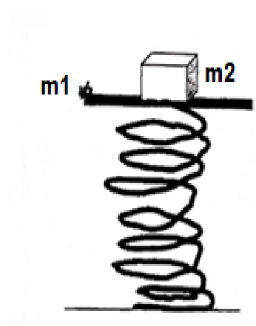
c. The mass is pulled a distance d from the point of equilibrium.
What is $d_{\max}$, the maximum distance which the mass can be pulled, such that the tension does not reset throughout the motion?

8) Rod on Two Wheels.



A rod of mass M is placed on two wheels which are fixed at their center. The wheels spin with angular velocity ω such that the right wheel spins anticlockwise and the left wheel spins clockwise. Between the rod and the wheels there is known kinetic friction. The rod is placed such that the center of the rod is a distance A from the midpoint between the two wheels.
Find the rod's frequency of oscillation.

9) Mass on a Surface on a Verticle Spring.



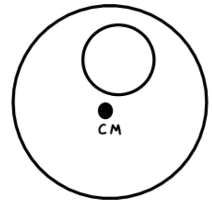
On a spring, of constant k, rest a surface of Mass m_1. A mass m_2 is placed on the surface.
The spring is pressed down to a height of $\mathrm{V}y$ and is released.

a. At what value of $\mathrm{V}y_{\min}$ will m_2 break away from the surface?

b. If $\mathrm{V}y = 2\mathrm{V}y_{\min}$, $\text{k}=10\dfrac{Nt}{m}$, $m_1 = 0.04\text{kg}$, and $m2 = 0.06\text{kg}$,
at what moment does m_2 break away from the surface?

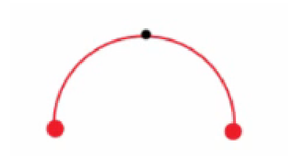
c. Given the values above, what is the position and velocity of the surface at the moment that m_2 breaks away?

10) Disk with a Hole.



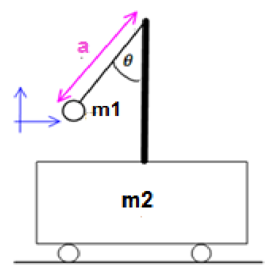
Find the frequency of the small oscillations of a disk with radius R, where a hole of size $R/4$ has been drilled a distance of $R/2$ from the center of the disk.
The disk has been nailed to a wall.
The mass of the disk, prior to having a hole drilled in, was M.

11) Half Hoop and Two Masses.



Find the frequency of the halved hoop, with radius R and mass M.
At each end of the hoop there is a mass, m.
The hoop is hung via a screw at its center.

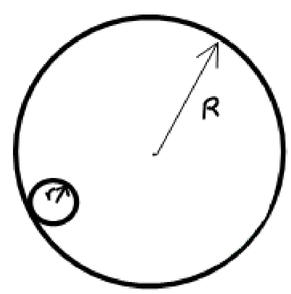
12) Pendulum On Moving Trolley.



A cart of mass m_2 is free to move on a frictionless horizontal plane.
On rod is attached perpendicularly to the cart and a mathematical pendulum with mass m_1 and length of string, a, is attached to the rod.
Mass m_1 is released from rest at a given angle of θ_0° (the cart is also at rest).

a. Find the velocity of the pendulum in the carts reference frame and a function of θ and θ'.
b. Find the velocity of the cart and the pendulum as a function of of θ and θ'.
c. Write an equation for the system's conservation of mechanical energy.
d. Write the equation for the conservation of energy at small angles.
e. What is the frequency of oscillations of m_1?

13) Disk Rolling in Pipe.



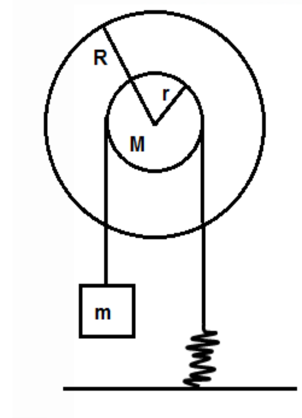
A disk of radius r is rolling up and down a pipe's inner wall, of radius R. The pipe is attached to the ground.

a. What will be the frequency of the oscillations of the disk, given that there isn't friction?
b. What will be the answer to a., if the friction was added to the pipe's walls (no slipping)?
c. What will be the answer to a., if, as well as friction on the floor being added, a further frictional force, $F = -bv$, would be added?

14) Double Disk, Mass and Spring.

A disk is nailed at its center to the wall.
The disk comprises of two disks stuck together, the smaller disk with a radius r, and the larger disk with a radius R.
Strings are wrapped around the disks.
The strings don't slip.

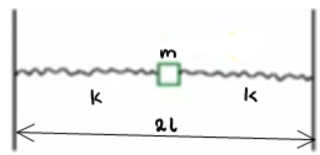
a. What is the frequency of oscillation?
b. What is the energy of the system?



Advanced Exercises

15) Mass between Two Springs.

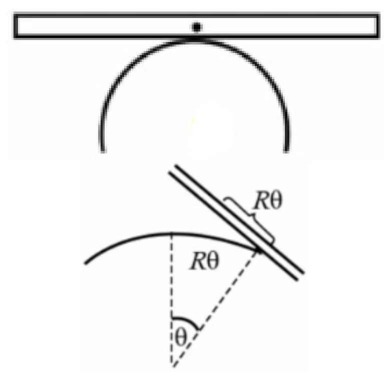
Between two walls, a distance of of $2L$ from one another, there is a mass, m, which is attached to the walls by springs, with constants k and resting length, L_0.
Find the frequency on the y-axis.



16) Rod on Half Sphere.

A rod of length l and mass m is resting on a ball of radius R.

a. Find the frequency of the rod.
b. Find the height of the rod's center of mass as a function of its angle.



Answer Key:

1) a. $x = \frac{m_2 g}{k}$ b. $\omega = \sqrt{\frac{k}{m_1 + m_2}}$ c. $A_{max} < \frac{g}{\omega^2}$

2) a. b.

3) $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{9}{L}t}\right)$

4) Refer to the video.

5) $y(t) = \frac{k\frac{y_0}{m}}{\frac{k}{m} - \omega^2}\cos(\omega t) + y_0^{'}$

6) a. $\omega = \sqrt{\frac{k}{n+M}}$ b. $x(t) = D\cos\left(\sqrt{\frac{k}{n+M}}t\right) + \frac{Mg}{k}$

7) a. $x_0 = m_1 \frac{g}{k}$ b. $\omega = \sqrt{\frac{k}{m_1 + \frac{1}{2}m_2}}$ c. $d_{max} = \frac{m_1 g}{k}$

8) $\omega = \sqrt{\frac{\mu_k g}{d}}$

9) a. $\Delta y_{min} = \frac{(m_1 + m_2) g}{k}$ b. $t_1 = \frac{1}{\omega}\cos^{-1}\left(-\frac{1}{2}\right)$ c. $v(t_1) = 2\Delta y_{min}\,\omega \sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

10) $\omega^2 = -\left(\frac{16}{247}\frac{g}{R}\right)$

11) $\omega^2 = -\left(\frac{m\%gb}{I}\right)$

12) a. $v_x = \dot{\theta} a \cos\theta, \quad v_y = \dot{\theta} a \sin\theta$

b. $v_T = -\frac{m_1}{m_2} v_P, \quad v_P = \frac{\dot{\theta} a \cos\theta}{1+\frac{m_1}{m_2}}, \quad v_{T.y} = 0, \quad v_{P.y} = v'_{P.y}$

c. Refer to the video.

d. $E(\theta) ; \frac{1}{2} m_1 \left[\frac{\dot{\theta} a^2}{1+\frac{m_1}{m_2}} - 2ga + ga\theta^2 \right]$

e. $\omega = \sqrt{\frac{g\left(1+\frac{m_1}{m_2}\right)}{a}}$

13) a. $\omega = \sqrt{\frac{g}{R}}$

b. $\omega = \sqrt{\frac{2g}{3R}}$

c. $\omega' = \sqrt{\frac{2g}{3R} - \left(\frac{b}{2m}\right)^2}$

14) a. $\omega^2 = \frac{KR}{A}$

b. $E_{total} = \frac{1}{2} kx^2 - mgx - 1\frac{1}{2} I\omega^2 + \frac{1}{2} m\dot{x}^2$

15) $\omega^2 = -\frac{2k(L-L_0)}{mL}$

16) a. $\omega^2 = -\frac{12gR}{L^2}$

b. $y_{cm} = R\left(1+\frac{\theta^2}{2}\right)$