

Workbook



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Vectors

Introduction to 2D and 3D Vectors

Questions

- 1) Find each of the following 2D/3D vectors, its magnitude, and whether it's a unit vector:
- The displacement vector from $(-8, 3)$ to $(5, -2)$.
 - The displacement vector from $(2, 3, 4)$ to $(2, 4, 4)$.
 - The position vector for $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.
 - The position vector for $(-8, 3, 5)$.
 - The vector $\vec{v} = \langle 7, -3, 0 \rangle$ starts at point $P(-3, 4, -1)$. At what point does it end?

Answer Key

- 1) a. $(13, -5)$ not unit vector b. $(0, 1, 0)$ is unit vector c. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is unit vector
d. $(-8, 3, 5)$ e. $(4, 1, -1)$

Vectors

Vector Arithmetic

Questions

- 1) Given the 2D vectors $\vec{a} = \langle 7, 4 \rangle$ and $\vec{b} = \langle -2, 5 \rangle$, compute the following:
- $5\vec{a}$
 - $6\vec{b} - 3\vec{a}$
 - $\|9\vec{a} + 4\vec{b}\|$
- 2) Given the 3D vectors $\vec{u} = 7\vec{i} - 2\vec{j} + 4\vec{k}$ and $\vec{v} = 6\vec{j} - 2\vec{k}$, compute the following:
- $-4\vec{v}$
 - $10\vec{u} + \vec{v}$
 - $\|-8\vec{u} - 3\vec{v}\|$
- 3) Answer the following questions:
- Find a unit vector that points in the same direction as $\vec{v} = \vec{i} - 4\vec{j} + 8\vec{k}$.
 - Find a vector that points in the same direction as $\vec{w} = \langle -2, 5 \rangle$ with a magnitude of 10.
- 4) Determine if the following pairs of vectors are parallel.
- $\vec{v} = 6\vec{i} - 4\vec{j} - 16\vec{k}$ and $\vec{w} = 15\vec{i} - 10\vec{j} - 40\vec{k}$.
 - $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 6, -4, 7 \rangle$.

Answer Key

1) a. $(35, 20)$ b. $(-33, 18)$ c. $\sqrt{6161} = 78.492$

2) a. $-24\vec{j} + 8\vec{k}$ b. $70\vec{i} - 14\vec{j} + 38\vec{k}$ c. $\sqrt{3816} = 61.77$

3) a. $\vec{v} = \frac{1}{9}\vec{i} - \frac{4}{9}\vec{j} + \frac{8}{9}\vec{k}$ b. $\left(-\frac{20}{\sqrt{29}}, \frac{50}{\sqrt{29}}\right)$

4) proof.

Vectors Dot Product

Questions

1) In each of the following, find the dot product $\vec{a} \cdot \vec{b}$:

a. $\vec{a} = \langle 5, -4 \rangle$ and $\vec{b} = \langle 4, 3 \rangle$.

b. $\vec{a} = 8\vec{i} + 6\vec{j} - 3\vec{k}$ and $\vec{b} = 6\vec{i} - 4\vec{j} + 7\vec{k}$.

c. $\|\vec{a}\| = 4$, $\|\vec{b}\| = 3$, and the angle between the two vectors is $\frac{\pi}{3}$.

2) In each of the following, find the angle between the two vectors:

a. $\vec{a} = \langle 3, 5 \rangle$, $\vec{b} = \langle 7, 6 \rangle$.

b. $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{w} = 5\vec{i} + 6\vec{j} - 7\vec{k}$.

3) In each of the following, determine if the two vectors are parallel, orthogonal or neither:

a. $\vec{p} = \langle 1, -2, 3 \rangle$, $\vec{q} = \langle 5, -8, -7 \rangle$.

b. $\vec{a} = \langle 3, 5 \rangle$, $\vec{b} = \langle 7, 6 \rangle$.

c. $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{w} = -5\vec{i} + 10\vec{j} - 15\vec{k}$.

4) Compute $proj_{\vec{u}}\vec{v}$ [projection of \vec{v} onto \vec{u}] for the following pairs of 2D/3D vectors:

a. $\vec{u} = \langle 4, -1 \rangle$, $\vec{v} = \langle 1, 7 \rangle$.

b. $\vec{u} = 7\vec{i} - \vec{j} + \vec{k}$, $\vec{v} = -2\vec{i} + 5\vec{j} - 6\vec{k}$.

5) Find the direction cosines and direction angles for $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$.

Answer Key

- 1) a. 8 b. 3 c. 6
- 2) a. $\theta = 18.435^\circ$ b. $\theta = 135.521^\circ$
- 3) a. Orthogonal b. Neither c. Parallel
- 4) a. $\left(-\frac{12}{17}, \frac{3}{17}\right)$ b. $\left(-\frac{161}{51}, \frac{23}{51}, -\frac{23}{51}\right)$
- 5) Solution is in the video.

Vectors Cross Product

Questions

1) Answer the following questions:

- Given $\vec{a} = \langle 3, -2, 5 \rangle$ and $\vec{b} = \langle 6, -4, 7 \rangle$. Compute $\vec{a} \times \vec{b}$ and use the result to find $\vec{b} \times \vec{a}$.
- Given $\vec{u} = 3\vec{i} - \vec{j} + 5\vec{k}$ and $\vec{v} = 4\vec{j} - 2\vec{k}$. Compute $\vec{u} \times \vec{v}$ and use the result to find $\vec{v} \times \vec{u}$.

2) Find a vector that is orthogonal to the plane containing the points P(1,2,3), Q(6,5,4) and R(7,8,9).

3) For each of the following sets of 3 vectors, determine if they lie in the same plane or not:

- $\vec{a} = \langle 3, -2, 5 \rangle$, $\vec{b} = \langle 6, -4, 7 \rangle$, $\vec{c} = \langle 1, 0, 1 \rangle$.
- $\vec{u} = \vec{i} + 4\vec{j} - 7\vec{k}$, $\vec{v} = 2\vec{i} - \vec{j} + 4\vec{k}$, $\vec{w} = -9\vec{j} + 18\vec{k}$.

Answer Key

- 1) a. $(6, 9, 0), (-6, -9, 0)$ b. $-18\vec{i} + 6\vec{j} + 12\vec{k}$, $18\vec{i} - 6\vec{j} - 12\vec{k}$
- 2) $\langle 12, -24, 12 \rangle$
- 3) a. Not in the same plane. b. In the same plane

The 3D Coordinates System

Questions

- 1) Find the projection of the point $P(4, 7, -5)$ onto the three coordinate planes.

- 2) Answer the following questions:
 - a. What is the distance of the point $P(4, 7, -5)$ from the z -axis?
 - b. Which point is closer to the z -axis: $P(4, 7, -5)$ or $Q(5, -6, 7)$?

- 3) Answer the following questions:
 - a. What is the distance of the point $P(4, 7, -5)$ from the xy -plane?
 - b. Which point is closer to the xy -plane: $P(4, 7, -5)$ or $Q(5, -6, 7)$?

- 4) Answer the following questions:
 - a. Given the equation $2x + 3y = 6$,
 - i. In \mathbb{R}^2 this is the equation of a: _____
 - ii. In \mathbb{R}^3 this is the equation of a: _____
 - b. Given the equation $(x-1)^2 + y^2 = 9$,
 - i. In \mathbb{R}^2 this is the equation of a: _____
 - ii. In \mathbb{R}^3 this is the equation of a: _____

Answer Key

- 1) Onto xy : $(4, 7, 0)$. Onto xz : $(4, 0, -5)$. Onto yz : $(0, 7, -5)$.
- 2) a. 5 b. Point P is closer.
- 3) a. $\sqrt{65}$ b. Point Q is closer.
- 4) a.(i) Line a.(ii) plane b.(i) Circle b.(ii) Cylinder.

Equations of Lines

Questions

- 1) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the points $(-10, 4, 0)$ and $(1, -4, 2)$.
- 2) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the point $(-10, 4, 0)$ and is parallel to the line $x = 3 + 4t$, $y = -2 + 3t$, $z = -5t$.
- 3) Let l_1 be the line through points $(4, 1, -5)$ and $(2, 0, 9)$ and let l_2 be the line given by $\vec{r}(t) = \langle 5, 1 - 9t, -8 - 4t \rangle$. Are the lines l_1 and l_2 parallel, perpendicular or neither?
- 4) Let l_1 be the line given by $x = -7 + 12t$, $y = 3 - t$, $z = 14 + 8t$ and let l_2 be the line given by $\vec{r}(t) = \langle 8 + t, 5 + 6t, 4 - 2t \rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.
- 5) Let l_1 be the line passing through points $(-5, 0, 2)$ and $(13, -2, 1)$ and let l_2 be the line given by $\vec{r}(t) = \langle 3, -1 - t, 2 + 4t \rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.
- 6) Let l be the line given by $x = -7 + 12t$, $y = 3$, $z = 16 + 8t$.
 - a. Does l intersect the xy -plane? If so, where?
 - b. Does l intersect the xz -plane? If so, where?

Answer Key

- 1) $\langle -10+11t, 4-8t, 2t \rangle$ $x = -10+11t, y = 4-8t, z = -2t$ $\frac{x+10}{11} - \frac{y-4}{8} = \frac{z}{2}$
- 2) $\langle -10+4t, 4+3t, -5t \rangle$ $t = \frac{x+10}{4}, t = \frac{y-4}{3}, t = -\frac{z}{5}$ $\frac{x+10}{4} = \frac{y-4}{3} = -\frac{z}{5}$
- 3) Perpendicular
- 4) $v = \frac{-39}{73}$; $u = \frac{88}{73}$
- 5) $\left(3, -\frac{8}{9}, 1\frac{5}{9} \right)$
- 6) a. $(-31, 3, 0)$ b. Not intersection.

Equations of Planes

Questions

- 1) Find the equation of the plane through the points $P(0,1,1)$, $Q(1,0,1)$ and $R(1,-3,-1)$.
- 2) Find the equation of the plane passing through the point $(0,2,-1)$ and orthogonal to the line $\vec{r}(t) = \langle 5+t, 1+3t, 4t \rangle$.
- 3) Find the equation of the plane containing the point $(-7,3,9)$ and parallel to the plane $4x+8y-2z=37$.
- 4) Plane π_1 is given by $4x+8y-2z=10$ and plane π_2 is given by $2x+y+8z=11$. Are the planes π_1 and π_2 parallel, orthogonal or neither?
- 5) Plane π_1 is given by $2x-3y+4z=5$ and plane π_2 passes through points $(1,2,2)$, $(2,2,3)$ and $(-3,-2,-6)$. Are the planes π_1 and π_2 parallel, orthogonal or neither?
- 6) Plane π is given by $2x-y+3z=6$ and line l is given by $x=1-t$, $y=3t$, $z=1+t$. Do l and π intersect? If so, where?
- 7) Plane π is given by $x-y+z=3$ and line l is given by $\vec{r}(t) = \langle 5+2t, 1-5t, 3t \rangle$. Do l and π intersect? If so, where?
- 8) Two planes π_1 and π_2 are given by $-x+7y-2z=24$ and $-5x+6y+3z=-3$, respectively. l is the line intersection of the planes: $l = \pi_1 \cap \pi_2$. Find the vector equation of line l .
- 9) Plane π is given by $5x-3y-6z=4$ and line l is given by $\vec{r}(t) = \langle 5-10t, 1+6t, 12t \rangle$. Are l and π parallel, perpendicular or neither?

Answer Key

- 1) $2x + 2y - 3z = -1$
- 2) $x + 3y + 4z = 2$
- 3) $4x + 8y - 2z = -22$
- 4) Orthogonal.
- 5) Neither.
- 6) $\left(1\frac{1}{2}, -1\frac{1}{2}, \frac{1}{2}\right)$
- 7) Not intersect.
- 8) $\vec{r}(t) = \langle 33t, 2 + 13t, -5 + 29t \rangle$
- 9) Perpendicular.

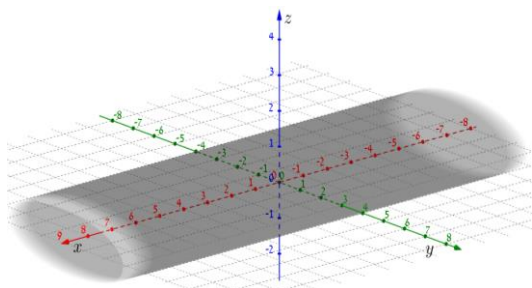
Quadric Surfaces

Questions

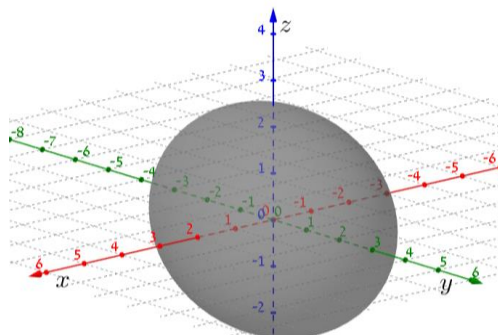
- 1) Sketch the graph of the quadric surface $\frac{y^2}{9} + z^2 = 1$.
- 2) Sketch the graph of the quadric surface $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{6} = 1$.
- 3) Sketch the graph of the quadric surface $z = \frac{x^2}{4} + \frac{y^2}{4} - 6$.
- 4) Sketch the graph of the quadric surface $y^2 = 4x^2 + 16z^2$.
- 5) Sketch the graph of the quadric surface $x = 4 - 5y^2 - 9z^2$.

Answer Key

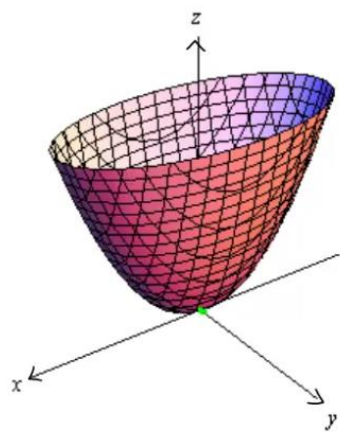
1)



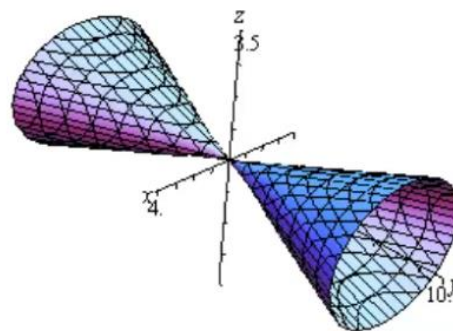
2)



3)



4)



5)

